Çankaya University – ECE Department – ECE 376

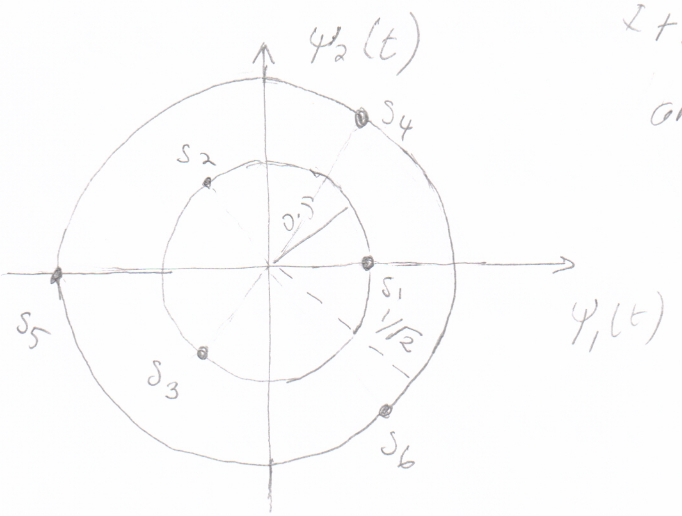
Student Name :
Student Number :

Open source exam
Duration : 2 hours

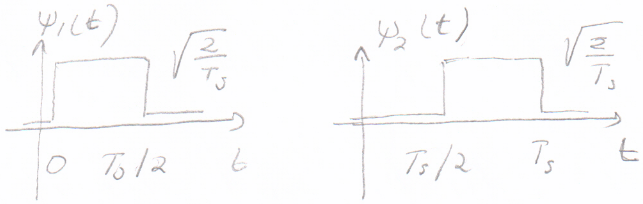
Questions

(70 Points) The six QAM signals, namely $s_1(t)$, $s_2(t)$, $s_3(t)$, $s_4(t)$, $s_5(t)$, $s_6(t)$ are placed as signal vectors $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$ on one circle and as signal vectors $\mathbf{s}_4$, $\mathbf{s}_5$, $\mathbf{s}_6$ on another circle. These circles are concentric (i.e. they are within each other and have the same centres) and signal vectors $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$, $\mathbf{s}_4$, $\mathbf{s}_5$, $\mathbf{s}_6$ are distributed on these two circles such that the end points of the signal vectors are at maximum distance from each other. Considering that the inner circle has a radius of 0.5 and the outer circle has a radius of $1/\sqrt{2}$, draw the constellation diagram of these QAM signals, writing for the $\mathbf{s}_{mn}$ coefficients of signal vectors, their lengths and energies. These six signal vectors then modulate six OFDM sub-carriers. Bearing in mind the orthogonality condition, determine the numerical values for the frequencies of these sub-carriers, if the symbol duration of the QAM signals is T_s = 1 msec. Draw the resulting time waveforms and the frequency spectrum, transmitter and receiver block diagrams. Explain how you would demodulate the sixth symbol from the modulated OFDM sub-carriers.

Solution: With the given data 6 QAM constellation is



If max distance criteria is sought and if for $\varphi_1(t)$ and $\varphi_2(t)$ are assumed to be



$s_1 = [0.5 \quad 0]$, $s_2 = [-0.25 \quad 0.433]$, $s_3 = [-0.25 \quad -0.433]$

$s_4 = [0.3536 \quad 0.6124]$, $s_5 = [-1/\sqrt{2} \quad 0]$, $s_6 = [0.3536 \quad -0.6124]$

In order to yield these coefficients, when multiplied by the basis functions $\varphi_1(t)$ and $\varphi_2(t)$, $s_i(t)$ would be

$$s_1(t) = \frac{1}{2}\sqrt{\frac{2}{T_s}} \quad 0 < t < T_s/2 \quad \text{or} \quad s_1(t) = \frac{1}{2}\varphi_1(t)$$

Lengths of $s_1, s_2, s_3 = 0.5$

Energies of $s_1, s_2, s_3 = 0.25 = (\text{Length})^2$

Lengths of $s_4, s_5, s_6 = 1/\sqrt{2}$

Energies of $s_4, s_5, s_6 = 0.5 = (\text{Length})^2$

Hence s_4, s_5, s_6 have energies twice of that in s_1, s_2, s_3.

Bearing in mind that when one symbol is placed on carrier, duration may be extended to $T = M \times T_s$ where $M = 6$ and $T_s = 1\,\text{mSec}$, thus $T = 6\,\text{mSec}$.

Now the first requirement that we should select subcarriers to have integer number of cycles within T, so for the first carrier we choose

$$f_1 = \frac{1}{T} = \frac{10^3}{6} = 166.67\,\text{Hz}$$

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Then $f_2 = 2 \times f_1 = 333.34$ Hz, $f_3 = 3 \times f_1 = 500$ Hz

$f_4 = 666.67$, $f_5 = 833.34$ Hz, $f_6 = 1$ kHz

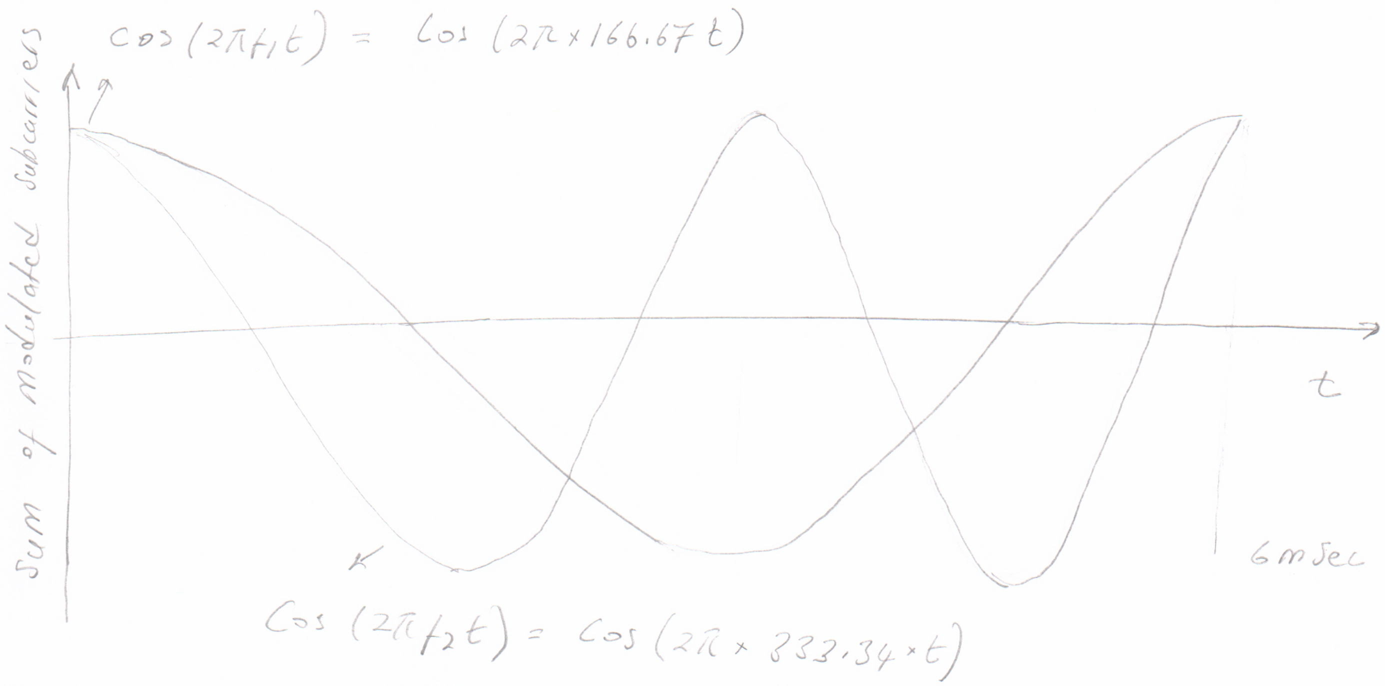
Another set would be (again satisfying orthogonality condition)

$f_1 = 1$kHz, $f_2 = 2$kHz ——— $f_6 = 6$kHz

Yet another possibility is to start with $f_1 = 1$kHz

but reduce subcarrier frequency spacing to 166.67 Hz

For the first case given, OFDM time signal



Other subcarrier similarly have an integer number of cycles within the duration of 6 mSec

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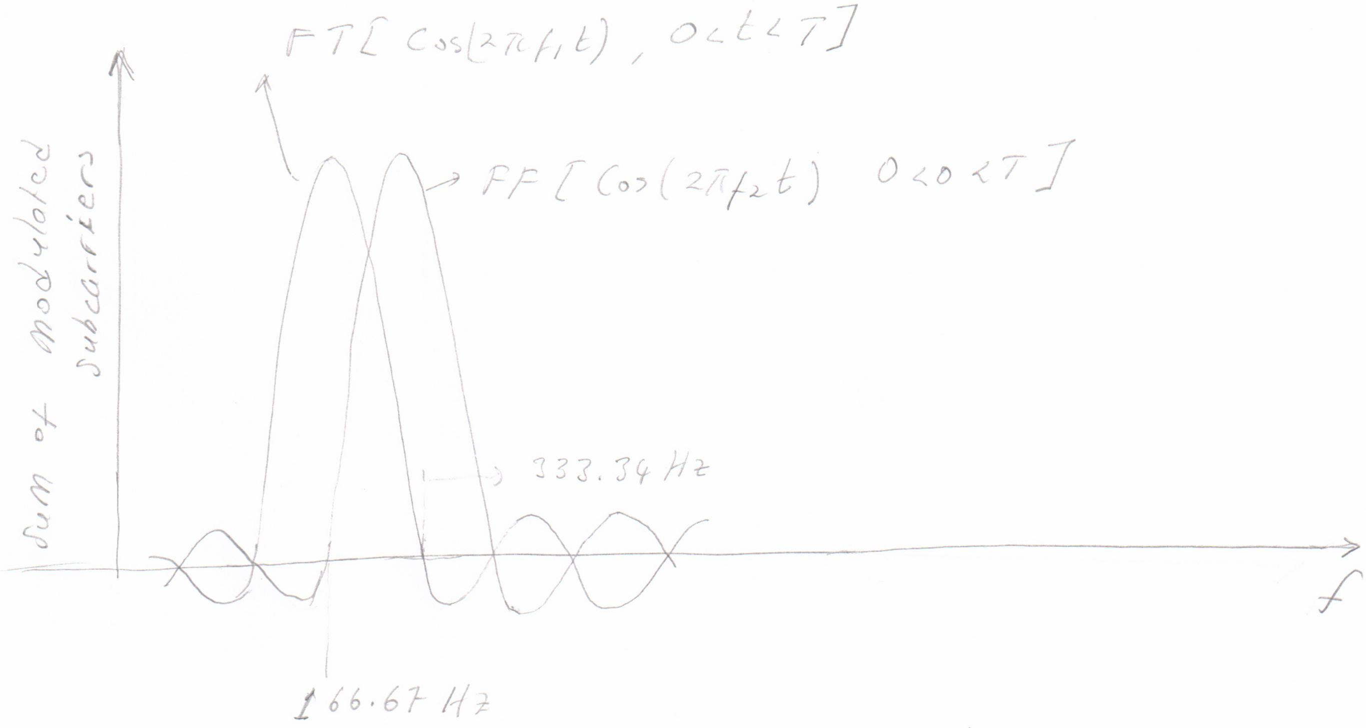
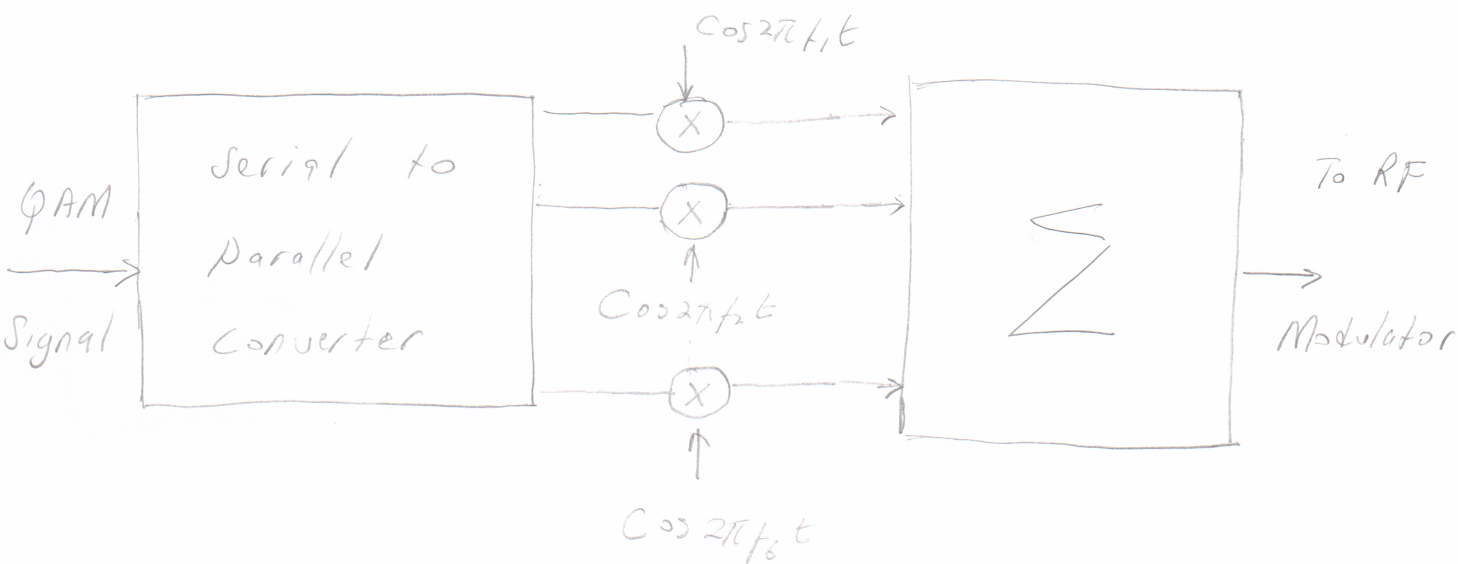
The spectrum of OFDM signal is shown below again taking the first case (one sided)



The spectrum of other (modulated) subcarriers similarly have have peaks at zero crossings of other subcarriers

Block diagram of transmitter



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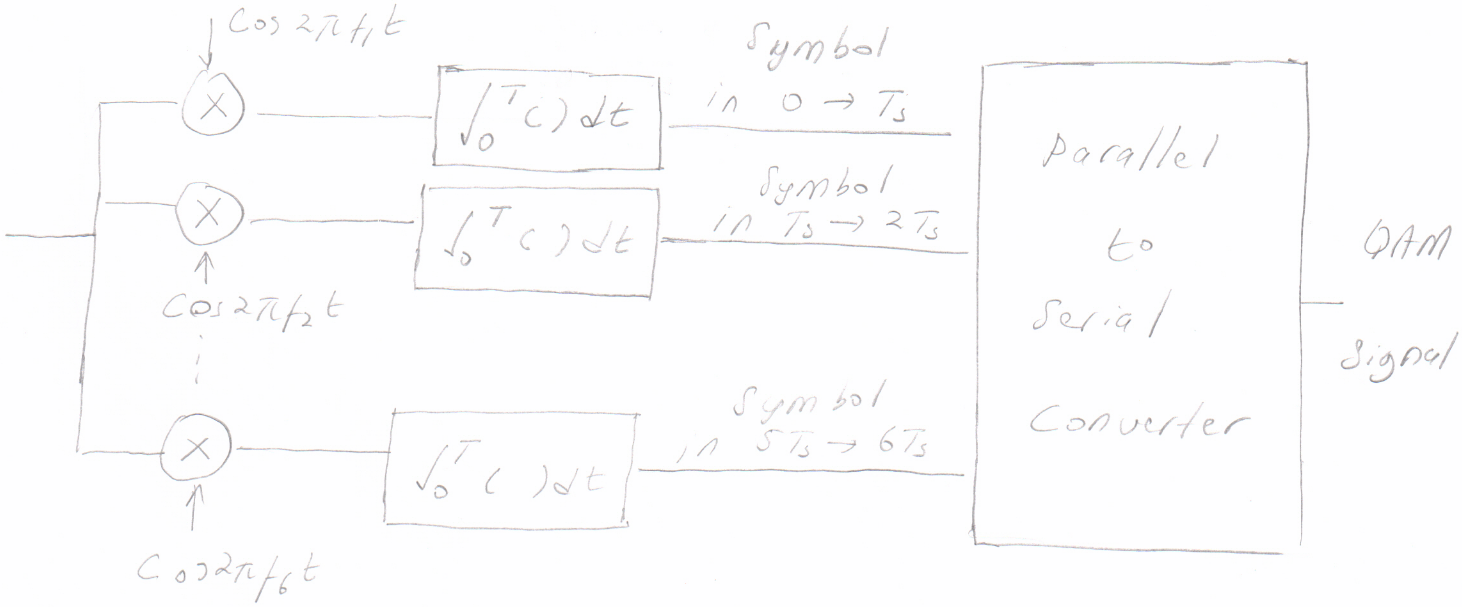
Block diagram of receiver



Note that which symbol is placed on which subcarrier depends on the stream of QAM signal

Assume that in the time interval of

$0 \to T_s \to 2T_s \quad \cdots \quad 5T_s \to 6T_s$

this stream is

$s_3 \quad s_1 \qquad\qquad s_4$

so this means we have to show how to demodulate s_4 on subcarrier f_6

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Transmitted OFDM signal is (in one T interval)

$$y(t) = \sum_{i=1}^{6} s_i \cos 2\pi f_i t$$

Normally this would arrive with noise at receiver and also there would be phase differences between subcarriers, we exclude these effects. At receiver. on each branch it is multiplied by the local carrier $\cos 2\pi f_p t$ and integrated over the interval $0 \rightarrow T$, so

$$I = \int_0^T \left(\sum_{i=1}^{6} s_i \cos 2\pi f_i t \times \cos 2\pi f_p t \right) dt$$

Essentially s_i is independent of time variable t therefore can be taken outside the integration

For a single term in the summation

I will become;

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$$I_i = \int_0^T \cos 2\pi f_i t \cos 2\pi f_p t \, dt$$

Convert into complex form and remember that transmitter implements inverse FT and receiver does the reverse therefore we label the transmitted subcarrier as $\exp(j 2\pi f_i t)$ and the (local) subcarrier of the receiver as $\exp(-j 2\pi f_p t)$, the I_i would be

$$I_i = \int_0^T \exp[-j 2\pi (f_i - f_p) t] \, dt$$

$$= -\frac{1}{j 2\pi (f_i - f_p)} \exp[-j 2\pi (f_i - f_p) t] \Big|_0^T$$

$= 0$ if $i \neq p$

$= T$ if $i = p$ (this is derived by going back to integral itself

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In the end I would become

$I = s_{i=p} T$, so the symbol corresponding to pth subcarrier would be demodulated

2. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) In DS spread spectrum system, a narrow band interference doe not affect our spread message signal : This depend on the SNR ratio of message signal energy to spectral density of narrow band interference signal, i.e $SNR = \frac{2E_b}{I_0}$ if this high no problem, I_0 is low if L_c is large

b) In 3G networks of Turkey, message signals are spread into a bandwidth of 5 k Hz : False

In DS spread spectrum systems, message signals are spread into a much wider bandwidth than the original bandwidth they occupy (reported to be 3.84 MHz in Turkey)

c) In OFDM, we choose sub-carriers to be orthogonal to each other, because orthogonality is the most bandwidth efficient method for multi-carrier multiplexing systems : True

we could have chosen so that subcarrier are well separated from each other, but this increases inefficiency. It is best to keep them apart at the limit of (time orthogonality)

d) In DS spread spectrum system, the transmitter sums all the spread message signals along time axis before transmission : True, since multiplexing (i.e separation of signals) is acheived along code axis.

e) At the receiver of DS spread spectrum system, demodulation is achieved by Matched Filter operation : True for demodulation we multiply the received signal by local PN and take the integral from 0 to T_b (for single user) detection.